CHAPTER 06 - Fully parametric analysis of dependence

6.1 Introduction

"In chapter 5 we reviewed some models representing the dependency of failure time on a vector z of explanatory variables. It is often useful to consider such models as formed from a baseline distribution holding under the standard conditions, z=0, plus a specification of the modification induced by non-zero z.

In the present chapter, we suppose that parts of the model are determined by a limited number of unknown parameters. This is called fully parametric.

For the analysis of fully parametric models, we concentrate on methods based on the likelihood function. Iterative numerical solution of the MLEs is nearly always involved and the availability of computer programs is crucial." (CO, 1984)

6.2 The likelihood (revisited)

"We now discuss a little more detail on the log-likelihood functions for common cases, concentrating on the special representation in which the function w(z;b) relating failure time T to failure time T_0 at z=0."
(Taken from CO, 1984)

We use equation (3.1) and put $p=\exp(b_0+b_1z_1+..)$
We then have at least 3 iterative schemes (see section 6.2.1). Estimation aims, described in section 3.1 apply, not just 1 iterative scheme, but to all iterative schemes.

6.2.1 Iterative schemes

l=d ln k+k(db_0+(b1*sum of z_{11} for those uncensored))+((k-1)(sum of ln x_1 for those uncensored))-exp(kb_0)(sum of exp(kb_1z_{11})*x_1^k (6.1)

Differentiate Eqn. (6.1) with respect to k,b_0,b_1 to get:-
dl/db_0 = kd-k*exp(kb_0)*sum of exp(kb_1z_{11})x_1^k (6.2)
dl/db_1 = k*sum of z_{11} for those uncensored-exp(kb_0)*sum of
z_{11}k*exp(kb_1z_{11})*x_1^k (6.3)
dl/dk = (d/k)+(db_0)+b_1*sum of z_{11} for those uncensored+sum of ln x_1 for those uncensored-(sum of (b_0+b_1z_{11}+ln x_1)*x_1^k*exp(kb_0+kb_1z_{11})) (6.4)

To obtain the iterative scheme for k, we rearrange Eqn. (6.4) and follow the similar steps as in Chapter 3. We finally obtain:-
k_{n+1}=1/(((b_0*exp(k_nb_0))*b_1*sum of z_{11}*exp($k_nb_1z_{11}$)*ln x_1*x_1^k(n)/d)-b_0-(b1*sum of z_{11} for those uncensored/d)-(sum of ln x_1 for those uncensored/d))

We use eqn. (6.2) and the new value of k to obtain b_0.
k_{n+1}d-k_{n+1}exp($k_{n+1}b_0$)*sum of exp($k_{n+1}b_1z_{11}x_1^k$(n+1). Put this to 0.
k_{n+1}d=k_{n+1}exp($k_{n+1}b_0$)*sum of exp($k_{n+1}b_1z_{11}x_1^k$(n+1).
d/sum of exp($k_{n+1}b_1z_{11}x_1^k$(n+1) = exp($k_{n+1}b_0$)
Take logs on both sides and rearranging to get
b_0 = (ln (d/sum of exp($k_{n+1}b_1z_{11}x_1^k$(n+1)) / k_{n+1}

We use eqn. (6.3) and new values of b_0 and k to obtain b_1.
Put this to 0 and rearrange this equation to get:-
$b_{1(n+1)}$=ln(exp($b_{1(n)}$)*sum of z_{11} for those uncensored)/sum of z_{11}*exp($k_{n+1}b_1z_{11}$)x_1^k (n+1))*exp($k_{n+1}b_{0(n+1)}$)

In F77 program 24, whose starting values are k=1.5, b_0=0.4 and b_1=0.25, I obtained k=1.26720, b_0=0.46234 and b_1=0.08562.
For meanings of $b_{0(n+1)}$,$b_{1(n)}$ and $b_{1(n+1)}$, see Appendix 01.
We can use these estimated parameters to obtain the elements of the information matrix, etc as in Chapter 3.
Equation (6.1) is for single regression. We can adapt equation (6.1) for multiple regression, by simply adding parameters b_2,b_3, etc. Then we differentiate these with respect to k, b_0,b_1, etc, put these to 0 and obtain iterative schemes.

Compared to Chapter 3, there are more implicit iterative schemes and hence more work (especially programming) in this chapter. However, iterative schemes in this chapter are more stable.

6.3 Computing

I used the following algorithm:-

(1) Simulate the Weibull distribution with parameters k,b_0,b_1,..
For each explanatory variable, I assumed that it was equally likely for the individual to be in some category, eg drug and generated random numbers in U(0,1). The individual is in some category when the random number is between 0.501 and 1.000. If this is so, I put the indicator variable=1 for some category.
(2) Obtain the sum of logs to the base e and the no. of individuals in each category for uncensored individuals.
(3) The old value of each parameter is set to the new value of each parameter.
(4) Obtain the new value of k.
(5) Use the new value of k to obtain b_0.
(6) Use the new values of k and b_0 to obtain b_1.
(7) Use the new values of k,b_0,... and b_{p-1} to obtain b_p.
(8) Do the old and new values of each parameter agree to a certain no. of decimal places? If not, reset all the sums, except those for uncensored individual and GOTO (3).

6.4 Summary

This chapter relates to most of my work in Chapters 3 and 5.
There are more implicit iterative schemes than those of Chapter 3. Of course, it means more work (especially programming). However, the iterative schemes are more stable than those of Chapter 3.